AD-753 402

FINAL TECHNICAL REPORT COVERING PERIOD 30 JUNE 1971 THRU 30 SEPTEMBER 1972

William H. Louisell

University of Southern California

Prepared for:

Army Missile Command

20 November 1972

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) University of Southern California	2a. REPORT SECURITY CLASSIFICATION
	2b. GROUP

3. REPORT TITLE

Final Technical Report
Contract #DAAHO1-71-C-1476

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)
Final Technical Report 30 June 1971 thru 30 September 1972

5. AUTHOR(S) (First name, middle initial, last name)

Dr. William H. Louisell

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
November 20, 1972	~~16~~ 34	1

8a. CONTRACT OR GRANT NO. DAAHO1-71-C-1476	9a. ORIGINATOR'S REPORT NUMBER(S)
b. PROJECT NO. D1-S-1800	
c.	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)
d.	

10. DISTRIBUTION STATEMENT

Distribution of this document is unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
	U.S. Army Missle Command Redstone Arsenal Alabama

13. ABSTRACT

AD 753 442

I

Final Technical Report

Contract # DAAH01-71-C-1476

November 20, 1972

The following report is a summary of work performed under the above contract relating to means of obtaining higher power output from CO_2 lasers.

A. Doppler Broadening

In an effort to study the effect of higher order transverse and longitudinal mode excitation on single mode power output from a CO_2 laser, Lax, Louisell and McKnight(1) [LLM] have shown that for a homogeneously broadened laser the fundamental mode, through non-linearities, will excite higher order modes. This means that atoms which would normally supply energy to the fundamental mode are wasted. It was therefore thought to be of interest to study the effect of Doppler broadening to see if the molecular motion could aid in suppressing higher order modes.

The problem of solving the coupled equations of motion for the atomic inversion, atomic polarization, and the amplitudes of the various excited transverse modes for atoms having a Maxwell-Boltzman velocity distribution is very difficult. The usual technique of Fourier analyzing the variables in time harmonics works if the atoms are either 1) motionless, or 2) moving through a spatially harmonic field (i.e., moving in the z direction through purely longitudinal modes). The technique does not work for atoms moving through the Hermite-Gaussian fields of the various transverse modes of spherical resonators.

(1) M. Lax, W.H. Louisell and W.B. McKnight, Jour. App. Phys., 43 3136 (1972).

The lasers of interest at Redstone Arsenal, however, are of fairly high pressure and it is easy to show that for pressures on the order of 70 torr (10^5 dynes/cm^2), the mean free path for any molecule is small compared to the wavelength of the laser radiation (10.6μ). The mean free path is

$$\ell \cong \frac{1}{\sqrt{2}\, n\sigma_0} = \frac{kT}{\sqrt{2}\, p\sigma_0} \, ,$$

where $\sigma_0 = \pi d^2$ is the cross section for scattering. At room temperature and 76 torr, we have

$$\ell \cong \frac{(1.4 \times 10^{-16})\ (3 \times 10^2)}{\sqrt{2} \times 10^5 \times \pi \times (2 \times 10^{-8})^2} \cong 2.4 \times 10^{-4} \text{ cm.}$$

The wavelength of the laser radiation is

$$\lambda = 10.6\mu \cong 10 \times 10^{-4} \text{cm.}$$

Therefore, at a pressure of 76 torr, the CO_2 molecules cannot move more than a wavelength before a phase-interrupting collision occurs. Hence, we may use the theory of Lax, Louisell and McKnight for homogeneously broadened lasers for those CO_2 lasers of interest at Redstone Arsenal since Doppler broadening will not assist in getting higher power output by supplying additional atoms to the fundamental mode region.

The homogeneous theory of LLM gave a criterion for suppressing higher order transverse modes. It showed that large fundamental mode volumes are desirable. Siegman and coworkers have been studying diverging mirrors to achieve this goal. However, these lead to undesirably large diffraction losses. As a compromise, we have studied mirrors which have diverging spherical mirrors in the central region of the beam surrounded by converging spherical mirrors. We expected that the fundamental mode volume would be increased by the diverging central part of the mirrors while the diffraction losses would be reduced by the converging rims of the mirrors. The following section gives the results of these studies.

B. Resonator Problem

The work on the resonator problem is now complete and the results may be found in the enclosed paper. The paper will be submitted shortly for publication. A talk was presented at the International Quantum Electronics Meeting in Montreal in May. The computer program now works well and a copy of the program has been sent to Dr. W. McKnight at Redstone Arsenal for any further geometries that need be studied. Dr. McKnight is in the process of checking the theory experimentally.

C. Two Mode Problem

To obtain a feel for how the power output of a multimode laser might compare to that of a single mode laser, we have attempted a study of a two mode laser. (Both modes are transverse modes.) We have been rather busy preparing the resonator paper for publication so that not too much progress has been made on the two mode problem. We have, however, been able to derive the coupled equations of motion for the amplitudes of the two modes, but we have essentially been limited by the complexity of the integrals obtained.

For the analysis of the two mode problem, we have proceeded along the lines of the (LLM) paper, only we have included two modes rather than one. We begin with their coupled equations of motion for the amplitude of any mode ℓ, and the inversion in the laser medium, D_M,

$$\dot{b}_\ell' + \left(i\omega_\ell - \frac{\gamma_\ell}{2}\right)b_\ell' = \sum_{M,m} U_\ell^*(\bar{r})U_m(\bar{r}) \frac{1}{P\left(\frac{d}{dt}\right)} (b_m' D_M) \tag{1}$$

$$D_M = D_W - \sum_{m,n} [U_m^* U_n \frac{1}{r\left(\frac{d}{dt}\right)} b_m'^{+} \frac{1}{P\left(\frac{d}{dt}\right)} b_n' + c.c.] L_M \qquad (2)$$

Here, b_ℓ', ω_ℓ, γ_ℓ and U_ℓ are the amplitude, cold cavity resonant frequency, loss due to diffraction and other loss mechanisms, and spatial distribution of the transverse mode ℓ, respectively, and each mode has taken on a factor μ, the atomic dipole moment, for simplicity. D_M is the time and space dependent atomic inversion in the cavity and D_W is the steady state, space independent inversion in the absence of any field. The operators,

$$r\left(\frac{d}{dt}\right) = \frac{\left(\frac{d}{dt} + \Gamma_1\right)\left(\frac{d}{dt} + \Gamma_2\right)}{2\frac{d}{dt} + \Gamma_1 + \Gamma_2} \qquad (3)$$

and

$$P\left(\frac{d}{dt}\right) = \Gamma + i\,\omega_a + \frac{d}{dt}$$

operate on all variables to their right. The original equations included another equation for the polarization in the medium, and the operators (3) appear in (1) and (2) because the polarization has been eliminated from the equations. In (3), Γ_1 and Γ_2 are the decay constants for the upper and lower atomic or molecular states a and b in Figure 1, ω_a is the resonant frequency and Γ is the decay constant for the polarization. In (1) and (2), the sums m and n are over all oscillating modes and the sum M is over all the atoms or molecules in the cavity and will ultimately be replaced by a volume integral.

We make the substitutions (4) into (1) and (2),

$$b_\ell' = b_\ell e^{-i\nu_\ell t}$$

$$D_M = \sum_\delta D_\delta^M e^{i\delta\Delta t} \qquad (4)$$

where ν_ℓ is now some frequency other than the cold cavity frequency of mode ℓ, and b_ℓ its slowly varying mode amplitude. Here, $\Delta = \nu_2 - \nu_1$ and D^M_δ depends only on position. In the equations resulting from substitution of (4) into (1) and (2), we match time harmonics and obtain the equations of motion for the amplitudes of the two oscillating modes and the amplitudes of the harmonics of the inversion, (all other modes have zero amplitude.)

$$\dot{b}_1 + [i\Delta_1 - \frac{\gamma_1}{2}]\, b_1 = \frac{1}{P[-i\nu_1]} \sum_M [\, |U_1|^2 b_1 D^M_o + U_2 U_1^* b_2 D^M_1] \tag{5}$$

$$\dot{b}_2 + [\, i\Delta_2 - \frac{\gamma_2}{2}]\, b_2 = \frac{1}{P[-i\nu_2]} \sum_M [\, |U_2|^2\, b_2 D^M_o + U_2^* U_1 b_1 D^M_{-1}] \tag{6}$$

$$D^M_o = D_W - D^M_o \left|\, |U_1|^2 |b_1|^2\, \Lambda_{11}(\delta) + |U_2|^2 |b_2|^2\, \Lambda_{22}(\delta) \right|$$

$$- D^M_1 (U_1^* U_2 b_1^+ b_2 \Lambda_{12}) - D^M_{-1} (U_2^*\, U_1 b_2^+ b_1 \Lambda_{21}) \tag{7}$$

$$D^M_1 = -D^M_1 \left|\, |U_1|^2 |b_1|^2 \Lambda_{11} + |U_2|^2 |b_2|^2 \Lambda_{22} \right|$$

$$- D^M_2 (U_1^* U_2 b_1^+ b_2 \Lambda_{12}) - D^M_o (U_2^* U_1 b_2^+ b_1 \Lambda_{21}) \tag{8}$$

$$D^M_{-1} = -D^M_{-1} (|U_1|^2 |b_1|^2 \Lambda_{11} + |U_2|^2 |b_2|^2 \Lambda_{22})$$

$$- D^M_o (U_1^* U_2 b_1^+ b_2 \Lambda_{12}) - D^M_{-2} (U_2^*\, U_1 b_2^+ b_1 \Lambda_{21}) \tag{9}$$

where,

$$\Lambda_{ij}(\delta) = \frac{1}{r(i\delta\Delta)} \left[\frac{1}{P(i\Delta\delta - i\nu_1)} + \frac{1}{P^*(i\delta\Delta + i\nu_o)} \right] \tag{10}$$

and

$$\Delta_i = \omega_i - \nu_i \qquad i = 1, 2. \tag{11}$$

The set (7) - (9) may be solved by perturbation theory assuming,

$$D_w > D^M_o > D^M_{\pm 1} > D^M_{\pm 2}, \text{ etc.}$$

and we obtain,

$$D^M_o = \frac{D_w}{1 + |U_1|^2 |b_1|^2 \Lambda_{11}(o) + |U_2|^2 |b_2|^2 \Lambda_{22}(o)} \tag{12}$$

$$D_1^M = \frac{-D_w\, U_2^* U_1\, b_2^+ b_1\, \Lambda_{21}(1)}{[1+|U_1|^2|b_1|^2\Lambda_{11}(0)+|U_2|^2|b_2|^2\Lambda_{22}(0)][1+|U_1|^2|b_1|^2\Lambda_{11}(1)+|U_2|^2|b_2|^2\Lambda_{22}(1)]} \tag{13}$$

$$D_{-1}^M = \frac{-D_w\, U_1^* U_2\, b_1^+ b_2\, \Lambda_{12}(-1)}{[1+|U_1|^2|b_1|^2\Lambda_{11}(0)+|U_2|^2|b_2|^2\Lambda_{22}(0)]\,[1+|U_1|^2|b_1|^2\Lambda_{11}(-1)+|U_2|^2|b_2|^2\Lambda_{22}(-1)]} \tag{14}$$

Then (13) in (5) yields,

$$0 = \dot{b}_1 + b_1 \Bigg\{ i\Delta_1 - \frac{\gamma_1}{2} - \frac{1}{P(-i\nu_1)} \left\langle \frac{D_w|U_1|^2}{1+|U_1|^2|b_1|^2\Lambda_{11}(0)+|U_2|^2|b_2|^2\Lambda_{22}(0)} \right\rangle$$

$$+ \frac{|b_2|^2}{P(i\nu_1)} \left\langle \frac{D_w|U_2|^2|U_1|^2\Lambda_{21}(1)}{[1+|U_1|^2|b_1|^2\Lambda_{11}(0)+|U_2|^2|b_2|^2\Lambda_{22}(0)]\,[1+|U_1|^2|b_1|^2\Lambda_{11}(1)+|U_2|^2|b_2|^2\Lambda_{22}(1)]} \right\rangle \Bigg\} \tag{15}$$

For the amplitude of mode 1 and we obtain a similar expression for the amplitude of mode 2 by substitution of (13) into (6). $\langle\ \rangle$ means average over the entire laser volume. The steady state number of photons in mode 1 is then given by

$$\frac{\gamma_1}{2} = \mathrm{Re}\Bigg\{ \frac{1}{P(-i\nu_1)} \left\langle \frac{D_w|U_1|^2}{(1+|U_1|^2|b_1|^2\Lambda_{11}(0)+|U_2|^2|b_2|^2\Lambda_{22}(0))} \right\rangle$$

$$+ \frac{|b_2|^2}{P(-i\nu_1)} \frac{D_w|U_2|^2|U_1|^2\,\Lambda_{21}(1)}{[1+|U_1|^2|b_1|^2\Lambda_{11}(0)+|U_2|^2|b_2|^2\Lambda_{22}(0)]\cdot[1+|U_1|^2|b_1|^2\Lambda_{11}(1)+|U_2|^2|b_2|^2\Lambda_{22}(1)]} \Bigg\} \tag{16}$$

with a similar expression for the steady state number of photons in mode 2. Similar expressions to (15) and (16) may be written for the frequencies of two modes, except that they involve the imaginary part of the same expression.

In order to compare the steady state number of photons in the two modes, it will be necessary to evaluate the spatial integrations over the modes U_1 and U_2 as denoted by the brackets $\langle\ \rangle$. This is where we ran into insurmountable difficulties in evaluating the complicated expressions. Some remarks about the "results" (15) and (16) are in order however.

1) If $|b_1|^2$ and $|b_2|^2$ are taken to be approximately constant, and the spatial integrations in (15) taken to be a constant, then (15) reduces to,

$$\dot{b}_1 + (A - \frac{\gamma_1}{2} + i\Delta_1)\, b_1 = 0 \tag{17}$$

where the definition of A is obvious but complicated. Equation (17) has the solution,

$$b_1 = b_o \exp\left|(\mathrm{Re}\, A - \frac{\gamma_1}{2})t + i(\Delta_1 + \mathrm{Im}\ A)t\right| \tag{18}$$

Now, b_1 will be stable (i.e. not grow or decay exponentially) if,

$$\frac{\gamma_1}{2} = \mathrm{Re}\, A$$

This little argument is evidently the source of (16) and uses the reasoning of (LLM).

2) The saturation terms come in as expected. In (LLM), for a single mode,

$$\frac{\gamma_1}{2} \propto \frac{1}{1 + \beta|b_1|^2}$$

and here we have,

$$\frac{\gamma_1}{2} \propto \begin{cases} \text{a term like } \dfrac{1}{1 + \beta|b_1|^2 + \gamma|b_2|^2} & \text{(a)} \\[2ex] \text{another term like } \dfrac{|b_2|^2}{[1+|U_1|^2|b_1|^2\Lambda_{11}(0)+|U_2|^2|b_2|^2\Lambda_{22}(0)]} \cdot \dfrac{1}{[1+|U_1|^2|b_1|^2\Lambda_{11}(1)+|U_2|^2|b_2|^2\Lambda_{22}(1)]} & \text{(b)} \end{cases}$$

Term (a) is like a total saturation term and (b) is like a cross saturation term for the effect of mode 2 on mode 1.

3) It seems strange to us that the steady state number of photons in mode 1 is defined in terms of the intensities $|b_1|^2$ and $|b_2|^2$ of both modes.

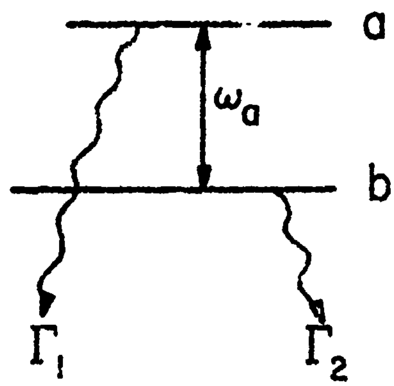


Fig. 1. Simple Laser Atomic or Molecular Model

Account DAAH02-71-C-1476 #53-4510-1641 Dr. W. H. Louisell

(August 1971 through June 1972)

TASK	Aug. '71 to May '72	June '72	Total	BUDGET	%	Free Balance	Outstanding Commitments
Travel	$ 3,029.06	$ 120.94	$ 3,150.00	$ 3,150.00	100	$ 0	$ 0
Communications & Xerox	14.23	260.55	274.88	264.00	104.1	< 10.88 >	0
Computer Time	541.60	100.31	641.91	1,050.00	61.1	408.09	358.09
M. & S.	36.00	0	36.00	36.00	100	0	0
Office Supplies	52.48	48.00	100.48	100.00	100.5	< .48 >	0
Wages	14,466.32	1,000.00	15,466.32	15,470.00	100	3.68	0
Overhead	6,365.18	440.00	6,805.18	6,807.00	100	1.82	0
Fringe Benefits	1,012.64	63.88	1,076.52	1,083.00	100	6.48	0
Total	$25,517.51	$2,033.58	$27,521.29	$27,960.00	98.5	408.71	358.09

TRANSVERSE MODE SUPPRESSION USING RIMMED UNSTABLE RESONATORS

by

M. Lax
City College of New York
New York, New York 10031

C. E. Greninger and W. H. Louisell
Departments of Physics and Electrical Engineering
University of Southern California
Los Angeles, California 90007

and

W. B. McKnight
U.S. Army Missile Command
Redstone Arsenal, Alabama 35809

ABSTRACT

We show that the criterion for single transverse mode laser operation can be reduced to the criterion that the intensity of the fundamental mode be greater than some constant in the entire laser volume. We also obtain amplitude and phase distributions for low loss, edge rimmed unstable resonators for Fresnel number up to eight. The resonators with the most uniform amplitude distributions are the most likely to suppress higher order transverse modes.

TRANSVERSE MODE SUPPRESSION USING RIMMED UNSTABLE RESONATORS

I. Introduction

There has been much interest in obtaining a high-power, high Fresnel number laser that will operate in the single lowest order transverse mode. In general, lasers will operate in numerous transverse modes simultaneously, or they will oscillate in a high order transverse mode. There have been a number of attempts to force the laser to oscillate in the lowest transverse mode. Birch[1] and Skinner and Geusic[2] have suggested the cats-eye-pinhole arrangement that spreads out the volume of the fundamental mode. In this way, the fundamental mode saturates out more of the atoms so that few atoms remain to excite the higher order modes. At high powers, however, it has been found that it is easy to destroy the pinhole.[2] It is therefore preferable to use no intra-cavity optics or apertures.

Siegman and Arrathoon[3] have studied unstable resonators in an attempt to spread out the mode volume of the fundamental mode and thus "discriminate" against higher transverse modes. The discrimination is based on the relative losses of the various modes and Siegman found the diffraction losses of even the fundamental mode to be quite high. Hence, it is possible to operate in a single transverse mode if high diffraction losses and high beam divergence can be tolerated.

Recently, Lax, Louisell, and McKnight[4] (LLM) have written down the coupled equations of motion for the mode amplitudes of a laser oscillating in a number of transverse modes and they have found a criterion for any higher order mode either to be quenched or driven into oscillation by a single strongly oscillating lowest order mode. They have evaluated this criterion for a confocal resonator and found that numerous higher modes would be driven into oscillation by the single lowest order mode.

In this paper, we show that the (LLM) criterion for the oscillation of only the fundamental mode may be reduced to the single criterion that the intensity distribution of the fundamental mode be greater than some constant over the entire laser volume. We then investigate the mode structure of unstable resonators with rims in order that we might obtain the low diffraction loss mode distribution that best satisfies this criterion, at least on the mirror surfaces. The unstable part of the resonator spreads out the mode volume, while the rims help to keep the energy from spilling out of the resonator.

It is found that by varying the resonator configuration one can find favorable intensity distributions at fairly low loss for Fresnel numbers from 2 to 8.6. It is also found that unless one is rather careful, he can easily force interior zeros in the intensity distribution across the mirrors. The appearance of zeros in the mirror intensity distribution becomes more sensitive to changes in mirror configuration at higher Fresnel number.

II. Special Case of (LLM) and the Rimmed Unstable Resonators

The analysis of (LLM) shows that a single oscillating fundamental mode will quench any other transverse mode if the quotient Q satisfies

$$Q = \frac{\gamma_E}{\gamma_\ell} \frac{\left\langle \frac{|u_\ell|^2 D_w}{K^2(0)} \right\rangle}{\left\langle \frac{|u_E|^2 D_w}{K(0)} \right\rangle} < 1 , \tag{1}$$

where γ_E, γ_ℓ and u_E, u_ℓ are the losses and spatial distributions of the strongly oscillating fundamental mode E and any other higher order mode ℓ, respectively. D_w is the pump distribution and

$$K(0) = 1 + S|u_E|^2$$

where S is a saturation factor that depends on the intensity of mode E and how close its frequency is to atomic resonance

$$S = \frac{2\mu^2 \Gamma |b_E|^2}{r(o) [\Gamma^2 + (\nu_E - \omega_o)^2]} .$$

Here, μ is the atomic dipole moment, b_E is the amplitude of mode E and ν_E is its frequency, ω_o is the atomic resonance frequency and r(o) is a combination of atomic decay constants. < > means integration over the volume of the active medium. (LLM) have evaluated (1) for a confocal resonator and have found that mode E would cause numerous higher order transverse modes to break into oscillation.

Without specifying the functional dependences of u_E and u_ℓ on $\vec{r}$, it is possible to reduce (1) to a simple expression. If $M = \max |u_E|^2$ and $C = \min |u_E|^2$, then (1) becomes

$$Q < \frac{D_w \gamma_E}{D_w \gamma_\ell} \frac{\left\langle \frac{|u_\ell|^2}{(1+SC)^2} \right\rangle}{\left\langle \frac{|u_E|^2}{(1+SM)} \right\rangle} = \frac{1+SM}{(1+SC)^2} < 1 , \tag{2}$$

where we have taken $\gamma_E \cong \gamma_\ell$ and D_w to be independent of position. Now, (2) will be satisfied if

$$C = \min_E |u|^2 > \text{some constant} \tag{3}$$

over the entire laser volume.

Equation (3) is a quantitative statement of the notion that one must spread out the mode volume of the single oscillating fundamental mode in order that it saturate the whole laser medium and leave no gain for the other modes.

In order to satisfy the criterion (3), at least over the mirrors, and also keep the diffraction losses to a minimum, we have investigated numerically the eigenvalues and eigenmodes of the type of resonator shown in Figure 1. The amplitude distribution that best satisfies (3) and keeps the diffraction losses to a minimum will be the one that is most uniform across most of the mirror and then falls rapidly to zero near the mirror edge.

III. Integral Equations

The resonator in Figure 1 is equivalent to an infinite lens system, a part of which is shown in Figure 2. The complex amplitude distributions $u_n(\vec{r})$ are defined in the plane immediately preceeding each lens. The distribution in front of lens n+1 that arises from the distribution in front of lens n, in the case where $d \gg |r|$, is,[5]

$$u_{n+1}(\bar{r}) = -\frac{i}{\lambda d} \int u_n(\bar{r}')\, e^{ik\rho(\bar{r},\bar{r}')} d^2 r' \,, \tag{4}$$

where the integration is over the surface preceding lens n. We make use of the following to reduce (4) to specific form:

1) The phase shift suffered by any ray in traversing the distance d, in a cylindrically symetric system, in the Fresnel approximation, is

$$k\rho_T(\bar{r},\bar{r}') = \sqrt{d^2 + (r'\cos\phi' - r\cos\phi)^2 + (r'\sin\phi' - r\sin\phi)^2}$$

$$\cong kd\left[1 + \frac{r^2 + r'^2 - 2rr'\cos(\phi - \phi')}{2d^2}\right]. \tag{5}$$

2) The phase shift associated with traversing lens n is,

$$k\rho_L(r') = \frac{kr'^2}{b'} \qquad r' \leq r_o$$

$$k\rho_L(r') = k\left(d - \frac{r'^2}{b}\right) \qquad r' \geq r_o \tag{6}$$

where α must be adjusted so that

$$\frac{r_o^2}{b'} = \alpha - \frac{r_o^2}{b}, \qquad \alpha = r_o^2\left[\frac{1}{b} + \frac{1}{b'}\right].$$

3) To form an integral equation, we let $u_{n+1}(\bar{r}) = \gamma u_n(\bar{r})$ where γ is a complex number that absorbs all uni-modual phase factors like e^{ikd} and i.

γ is related to the diffraction loss of the lens system per lens transit,

$$L = 1 - |\gamma|^2.$$

4) We stipulate that we are only interested in modes that are independent of ϕ. This permits us to do the ϕ' integration with the usual Bessel function as the result.

Equation (4) becomes the integral equation,

$$\gamma u(r) = \frac{k}{d}\int_0^{r_o} u(r')\, e^{\frac{i\pi}{d\lambda}\left[r^2 + (1+2d/b')r'^2\right]} J_o\left(\frac{k}{d}rr'\right) r'\,dr' \\ + \frac{k}{d}\, e^{ikd}\int_{r_o}^{r_1} u(r')\, e^{\frac{i\pi}{\lambda d}\left[r^2 + (1-2d/b)r'^2\right]} J_o\left(\frac{k}{d}rr'\right) r'\,dr' \tag{7}$$

where J_o is the zeroth order Bessel function. For convenience we reduce (7) to normalized form. Letting $x = \frac{r}{r_1}$, we obtain,

$$\gamma u(x) = 2\pi N1 \left\{ \int_0^q u(y)\, J_o(2\pi N1xy)\, e^{i\pi N1[x^2 + (1+2d/b')y^2]}\, ydy \right. \\ \left. + e^{2i\pi N0\left(\frac{d}{b} - \frac{d}{b'}\right)} \int_q^1 u(y)\, J_o(2\pi N1xy)\, e^{i\pi N1\left[x^2 + \left(1 - \frac{2d}{b}\right)y^2\right]}\, ydy \right\}, \tag{8}$$

where $q = \frac{r_o}{r_1} = \sqrt{\frac{N0}{N1}}$ and $N0 = \frac{r_o^2}{\lambda d}$ and $N1 = \frac{r_1^2}{\lambda d}$ are the Fresnel numbers of the inner and outer mirrors, respectively.

In terms of kernels, (8) is 18

$$\gamma u(x) = \int_0^q K_1(x,y)\, u(y) dy + \int_q^1 K_2(x,y)\, u(y)\, dy \tag{9}$$

where

$$K_1(x,y) = 2\pi N l J_o(2\pi N l x y)\, e^{i\pi N l \left[x^2 + \left(1+\frac{2d}{b}\right) y^2\right]} y$$

$$K_2(x,y) = 2\pi N l J_o(2\pi N l x y)\, e^{i\pi N l \left[x^2 + \left(1-\frac{2d}{b}\right) y^2\right]} y \, . \tag{10}$$

IV. Method of Solution

We solve the integral equation (9) by approximating the integrals on the right hand side of the equation by Legrendre-Gauss quadrature.[6] There, we represent the first integral by a sum of M terms and the second integral by a sum of N terms. In order to use this type of quadrature, we must transform the ranges of integration of both integrals to (-1,1). The details are rather tedious and will not be presented here. The integral equation (9) is thereby replaced by the set of algebraic equations,

$$\gamma u(x_i) = \sum_{j=1}^{M} K_1(x_i,x_j)\, B_j\, u(x_j) + \sum_{j=1}^{N} K_2(x_i,x_j)\, B_j\, u(x_j) \tag{11}$$

where $u(x_j)$ is the complex amplitude at unequally spaced mirror positions x_j. The x_j are related to the zeros of the Legendre polynomials $P_M(x)$ and $P_N(x)$, and B_j are tabulated weight functions.[7] Equation (11) is equivalent to the matrix equation,

19

$$\tilde{K}\bar{u} = \gamma \bar{u} ,$$

where $\tilde{K}$ is an $(M+N) \times (M+N)$ complex matrix whose elements are

$$K_1(x_i, x_j) B_j \qquad \text{for} \quad j \leq M$$

$$K_2(x_i, x_j) B_j \qquad \text{for} \quad j \geq M$$

and the vector $\bar{u}$ is the complex amplitude evaluated at the various x_j. The matrix elements are easily generated on computer and we then use the IBM SHARE subroutine ALLMAT to find the eigenvalues and right eigenvectors of the matrix $\tilde{K}$.[8] The right eigenvectors are then the eigenmodes of the cavity.

As a check on the mathematical formulation and the computer program, we have plotted the results of the special cases of plane parallel ($d/b = d/b' = 0$) and confocal ($d/b = 1$, $d/b' = -1$) resonators and compared them with the results of the wave launching method of Fox and Li in Figure 3.[9*] Evidently, the agreement with the old results is quite good and we may believe the results for all reasonable values of d/b and d/b'.

V. Results and Discussion

Having convinced ourselves that the computer program works for widely varied d/b and d/b', we can now proceed to investigate the behavior of the mirror amplitude distributions as a function of d/b and d/b'. In

*In figure 3 we have shown only the amplitude distributions, but the phase distributions compare equally well.

order to get some qualitative feel for how we expect the distributions to depend on the parameters, it is necessary to solve (8) in some approximate sense. Unfortunately, not only is the integral equation (8) composed of two parts, the parts are non-hermitian, non-symetric and non-degenerate. Furthermore, since we are interested in large as well as small Fresnel numbers, it is pointless to expand $J_0(2\pi N l xy)$ in a power series in its argument (the usual trick for obtaining a degenerate kernel).[10] We have spent considerable time trying to obtain any kind of approximate analytical solution to (8) and have been unsuccessful. Therefore, any qualitative feel for how the distributions will depend on the parameters is completely missing and we must proceed on a trial and error basis. Hence, it will be rather difficult to explain a logical progression to the desired results, and we will only be able to indicate how we obtained the results and hope to show their apparent importance. We begin with low Fresnel number.

A. Low Fresnel Number

In Figure 4 we show a series of mirror amplitude distributions for N1 = 2 and N0 = .25. (The intersection of the interior and exterior mirrors is indicated by the hash marks on the abscissas on all of the remaining figures. The diffraction loss of the lowest loss mode is L and that of the next lowest loss mode is occasionally listed in parenthesis.) This resonator is definitely of confocal-type with an interior spherical hump. We hold the "confocal parameter" d/b constant at .2 and steadily increase d/b' to force more of the energy to the outside of the resonator. The procedure seems to be working relatively well until an interior zero in the amplitude distribution begins to form and at $d/b' = 4.$ the distribution becomes quite unacceptable.

To remedy this situation, we went to the unstable-type resonator with rims shown in Figure 5 for $N1 = 2.$, $N0 = 1.5$. Here, the first attempt yielded an acceptable solution but the diffraction loss was rather high (25%/pass). Therefore, we increased d/b (made the rims more curved) to try to decrease the diffraction loss. At $d/b = .4$, $d/b' = .1$ we find a rather acceptable solution with a loss of 17% per pass. It is interesting to note that the amplitude ripples are of higher frequency and of lower amplitude for $N0 \cong N1$ (unstable with rims) and of lower frequency and higher amplitude for $N0 << N1$ (confocal with interior bumps). The unstable with rims are therefore of more interest to us because large amplitude variations across the mirror will undoubtedly cause zeros and the distribution will not satisfy the basic criterion (3). On the other hand, rapid amplitude variations are acceptable, provided they are of low modulation.

The phase distributions in either case did not change appreciably from run to run so we have plotted only a typical phase distribution in the lower right hand corners of Figures 4 and 5. Neither phase distribution varies appreciably over the entire mirror surface, but we do note that the phase varies more rapidly near the origin for $N0 \cong N1$.

B. Medium Fresnel Number

We found similar behavior for $N1 = 4.$ as for $N1 = 2.$ Namely, it was easier to find acceptable distributions for $N0 \cong N1$, shown in the left column of Figure 7, rather than for $N0 << N1$, shown in Figure 6. We also found that for N0 and N1 integer, the amplitude distribution could

22

have rather abrupt changes for small changes in d/b or d/b'. We feel that this high sensitivity to d/b and d/b' is probably due to the fact that for small d/b' the exponentials

$$e^{i\pi N1\left(1 + \frac{2d}{b'}\right)y^2}$$

and

$$e^{i\pi N1\left(1 - \frac{2d}{b}\right)y^2}$$

in the kernels in equation (8) oscillate over very nearly an integral number of half cycles over the ranges (0,q) and (q,1). We therefore felt that we would have more success for N0 and N1 not integers and this is borne out by the distributions shown in the right hand column of Figure 7. We feel that the second distribution in this column will be successful in suppressing higher order transverse modes at this Fresnel number. Note that the diffraction loss is only 4.5% per pass.

C. Higher Fresnel Number

In Figure 8 we have plotted the amplitude distributions for rather high Fresnel number, and we feel that the distribution at the bottom of the left column will be successful in suppressing higher order modes. Here, the distributions become very sensitive to changes in d/b and d/b' and for typical lasers a favorable distribution would probably require that the mirrors have radii of curvatures in the hundreds of meters. We feel, however, that provided the mirror is spherical to 1/20 th of a wave, that the distribution will be approximately the same even for large tolerances on the radius (i.e., 400 or 450 meters).

VI. Conclusions

Although the unstable resonators of Siegman and Arrathoon can be shown to have high discrimination against higher order transverse modes, this discrimination does not guarantee suppression of these higher modes. Indeed, as shown in (LLM), if the gain for a particular mode, caused by the presence of a strongly oscillating fundamental mode, exceeds the loss, the higher mode will break into oscillation, independent of the relative losses of the two modes. The analysis of (LLM), has provided us with a criterion for the suppression of any higher order mode by the fundamental mode, all gains included.

In this paper, we have (1) shown that this criterion can be reduced to a quantitative statement of the notion that one needs to spread out the mode volume of the fundamental mode in order to suppress higher order modes, and (2) solved numerically for the low diffraction loss eigenmodes of rimmed unstable resonators in order that we might find the mode amplitude distributions that best satisfy this criterion over the mirror surfaces.

24

References

1. J. M. Burch, "Design of Resonators", in Quantum Electronics, ed. P. Grivet and N. Bloembergen (Columbia University Press, New York, 1964), Vol. 2, pp. 1187-1202.

2. J. G. Skinner and J. E. Geusic, "A Diffraction Limited Oscillator", in Quantum Electronics, ed. P. Grivet and N. Bloembergen (Columbia University Press, New York, 1964), Vol. 2, pp. 1437-1444.

3. A. E. Siegman and R. Arrathoon, "Modes in Unstable Optical Resonators and Lens Waveguides", IEEE J. Quantum Electronics, Vol. 3, pp. 156-163, April 1967.

4. M. Lax, W. H. Louisell and W. B. McKnight, to be published in IEEE J. Quantum Electronics.

5. M. Born and E. Wolf, Principles of Optics, (Pergamon Press, New York, 1970), Ch. 8.

6. F. B. Hildebrand, Introduction to Numerical Analysis (McGraw-Hill, New York, 1965), Ch. 3.

7. C. H. Love, "Abscissas ans Weights for Gaussian Quadrature for N = 2 to 100 and N = 125, 150, 175 and 200", National Bureau of Standards Monograph 98 (1966).

8. R. L. Sanderson and W. Streifer, "Comparison of Laser Mode Calculations", Applied Optics, Vol. 8, pp. 131-136, Jan. 1969.

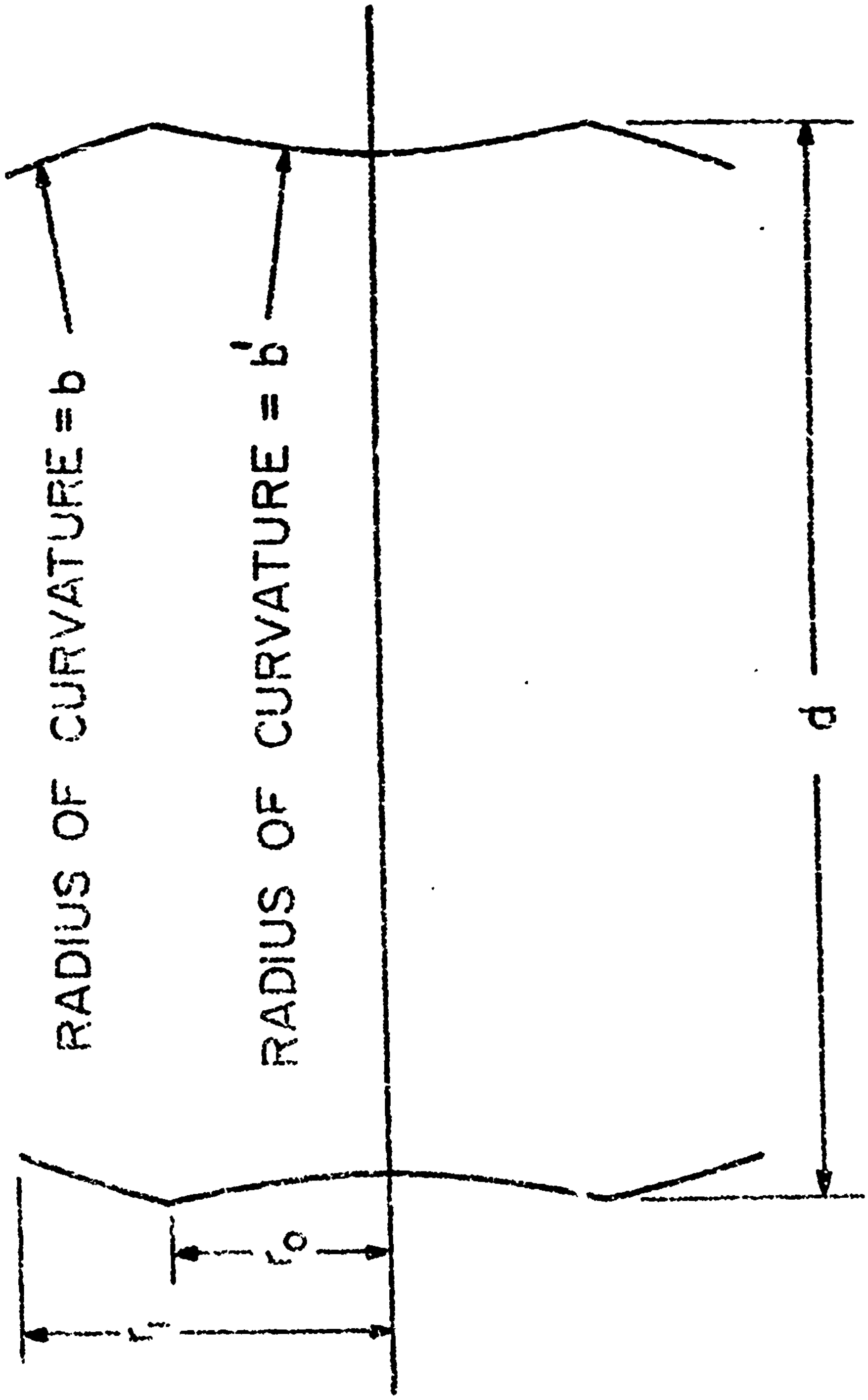
9. A. G. Fox and T. Li, "Resonant Modes in a Maser Interferometer", Bell Sys. Tech. J., Vol. 40, pp. 453-488, March 1961.

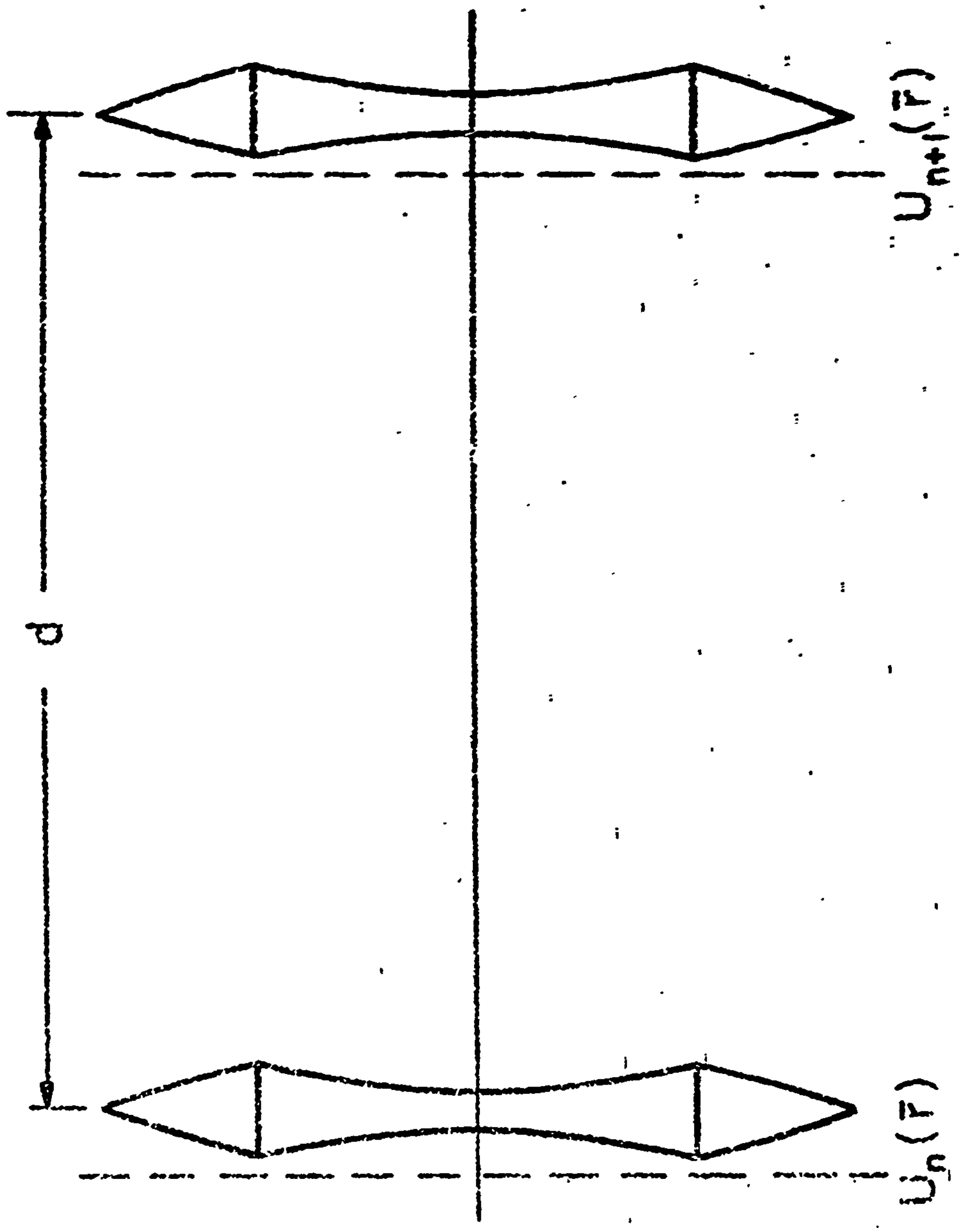
10. D. E. McCumber, "Eigenmodes of a Symmetric Cylindrical Confocal Laser Resonator and Their Perturbation by Output-Coupling Apertures", Bell Sys. Tech. J., Vol. 44, pp. 333-363, Feb. 1965.

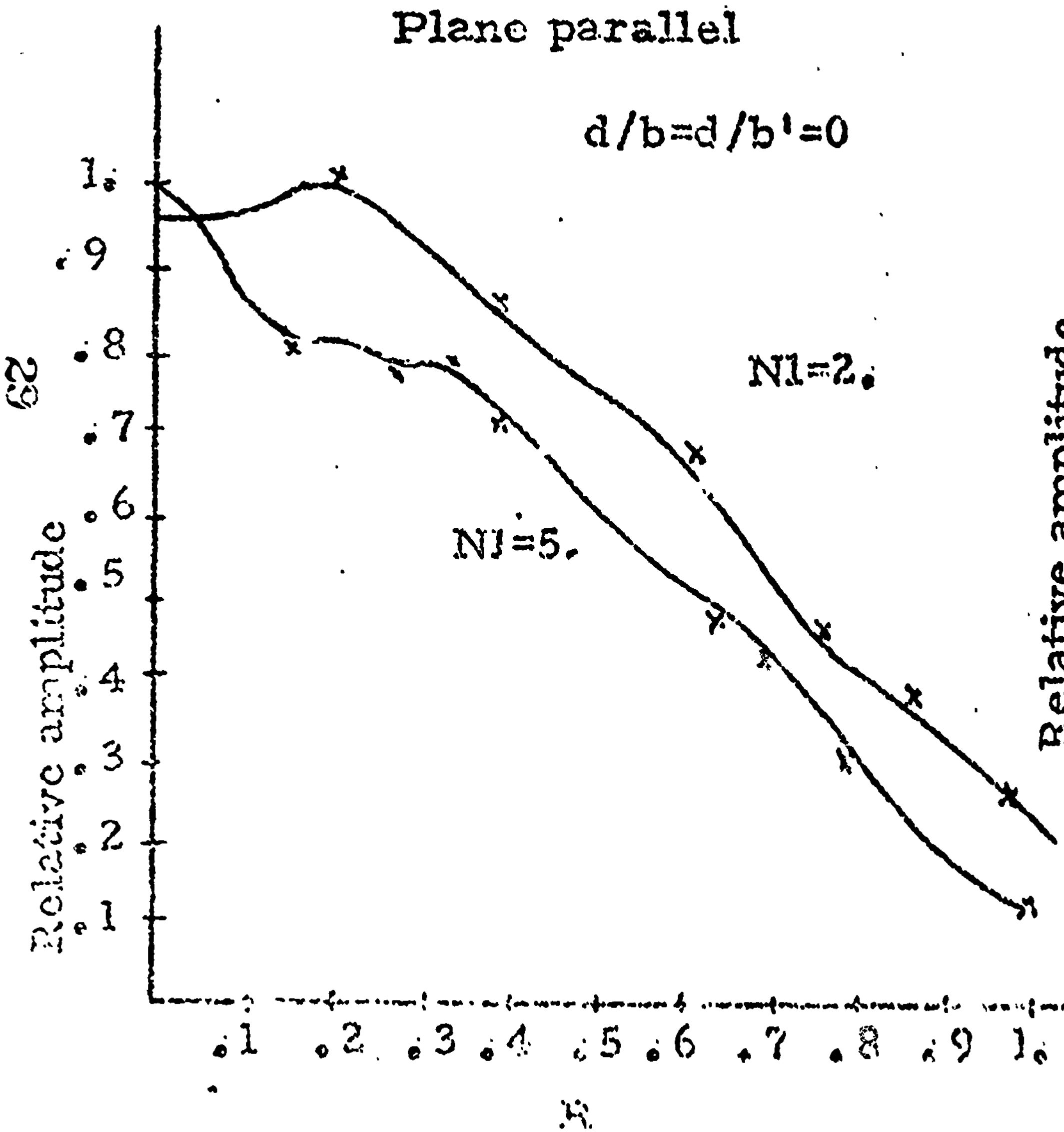
FIGURE CAPTIONS

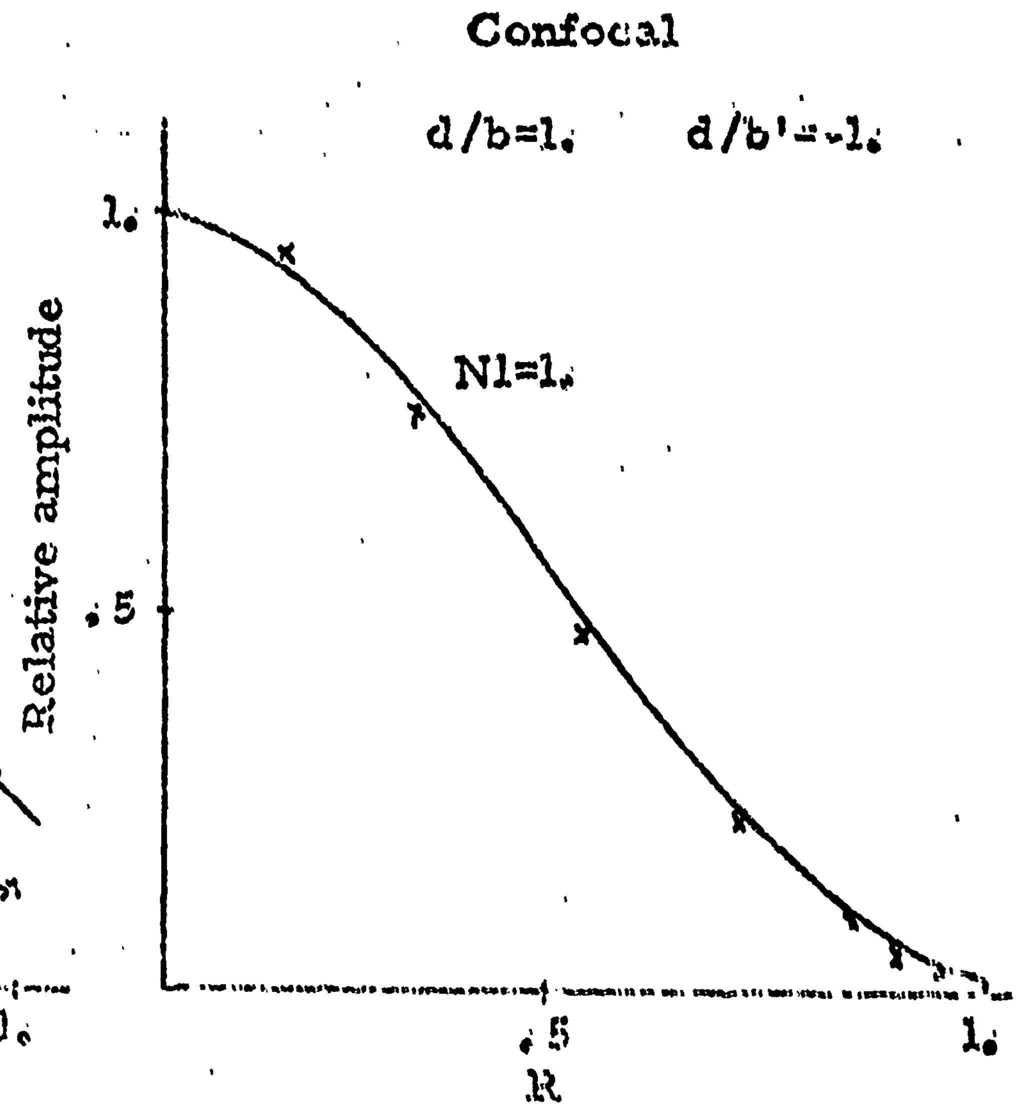
Figure 1	Unstable resonator with rims
Figure 2	Lens system that is equivalent to resonator shown in Figure 1
Figure 3	Comparison with Fox and Li wave launching method for plane parallel and confocal resonators. ———————— Fox and Li x x x x x x x x x Ours
Figure 4	Mirror amplitude and phase distributions for various d/b, d/b' at low Fresnel number. Confocal-type with interior spherical hump.
Figure 5	Mirror amplitude and phase distributions for various d/b and d/b' at low Fresnel number. Unstable with rims.
Figure 6	Mirror amplitude and phase distributions at medium Fresnel number. Confocal with interior spherical humps.
Figure 7	Amplitude distributions at medium Fresnel number for integer and non-integer Fresnel numbers. Unstable resonator with rims.
Figure 8	High Fresnel number amplitude and phase distributions. Unstable with rims.

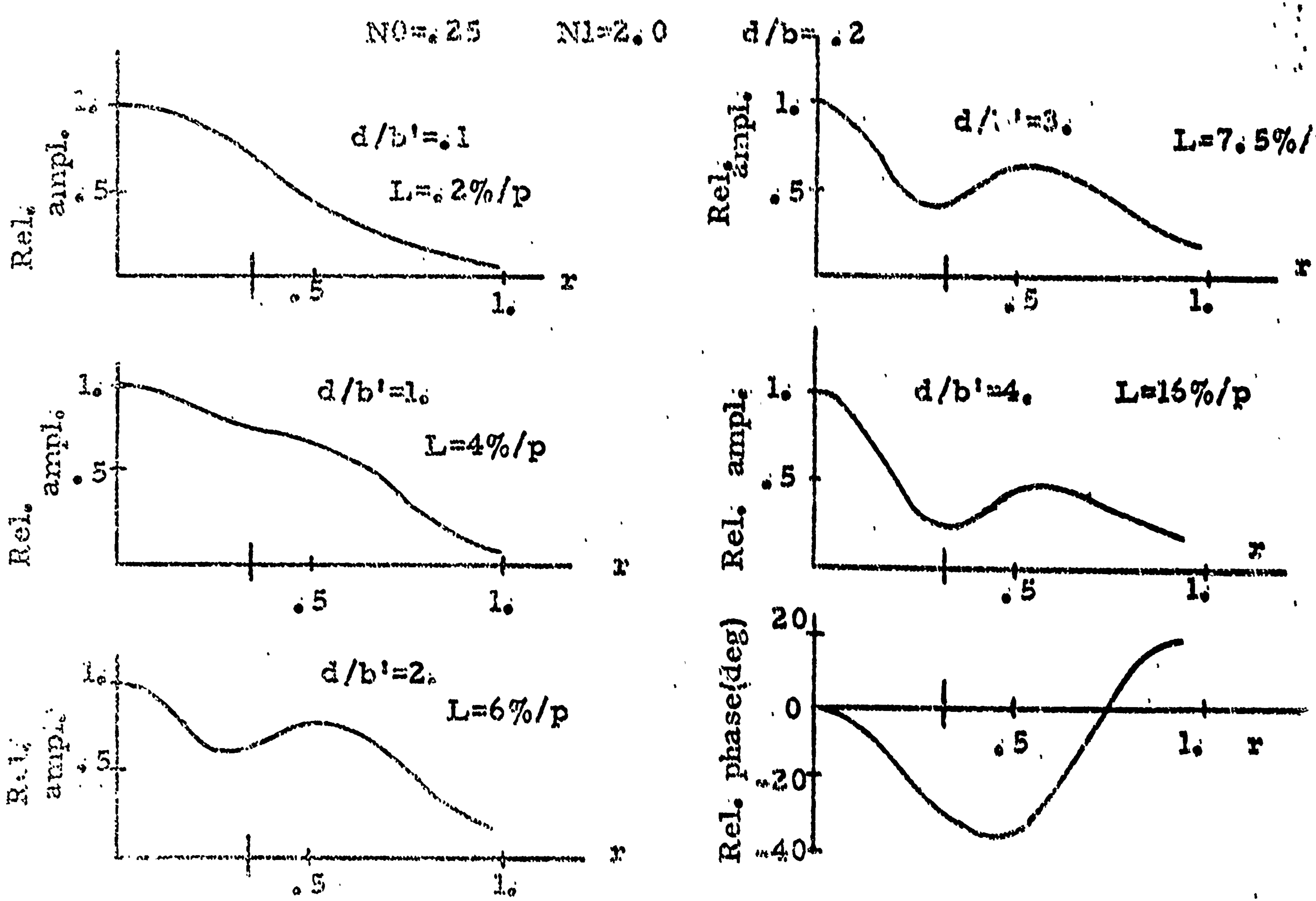
26


RADIUS OF CURVATURE = b
RADIUS OF CURVATURE = b'
d
r_o
r

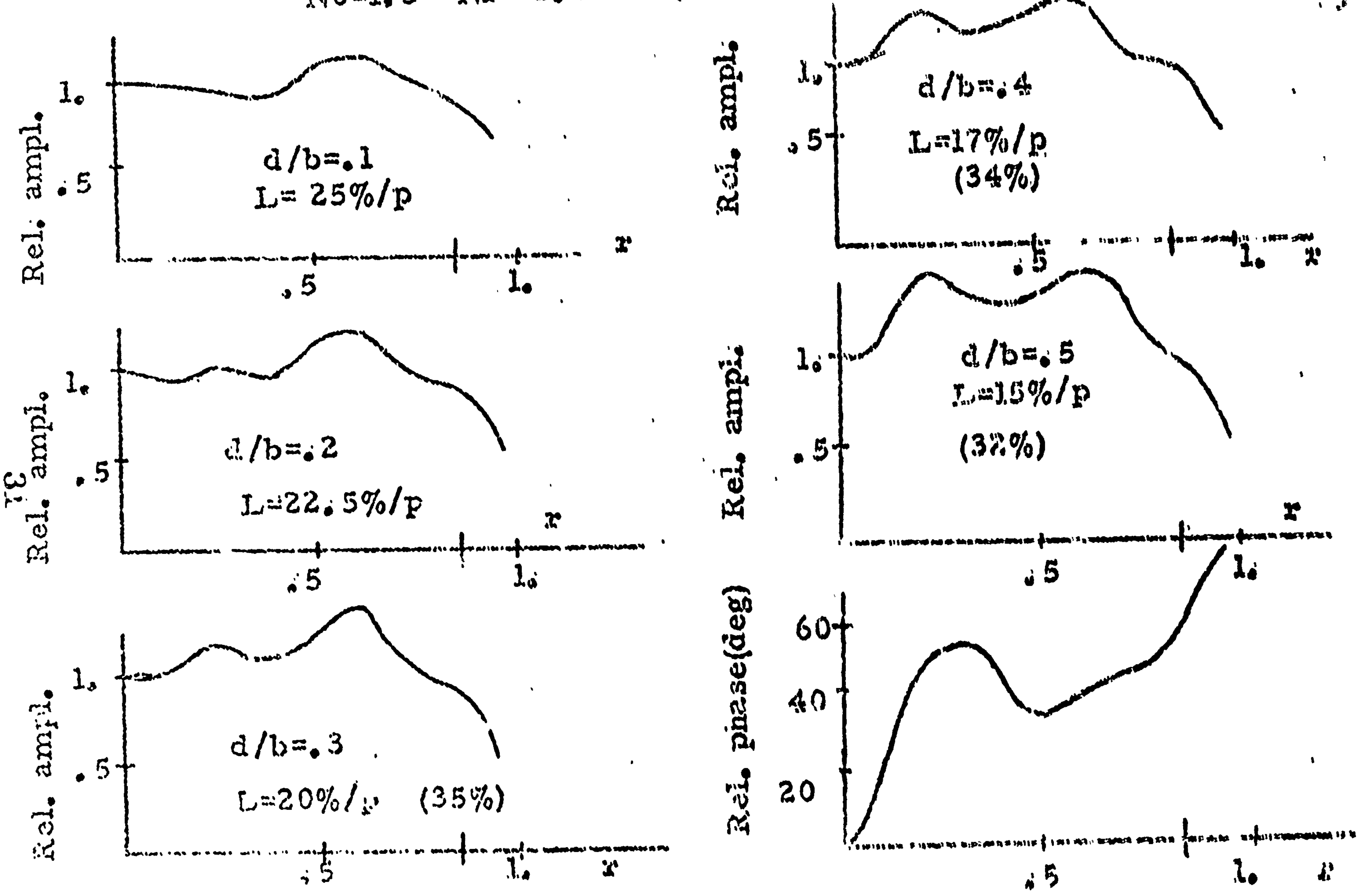

d
$U_{n+1}(r)$
$U_n(r)$


Plane parallel
d/b=d/b'=0
Nl=2.
Nl=5.
Relative amplitude
1.
.9
.8
.7
.6
.5
.4
.3
.2
.1
.1 .2 .3 .4 .5 .6 .7 .8 .9 1.
R

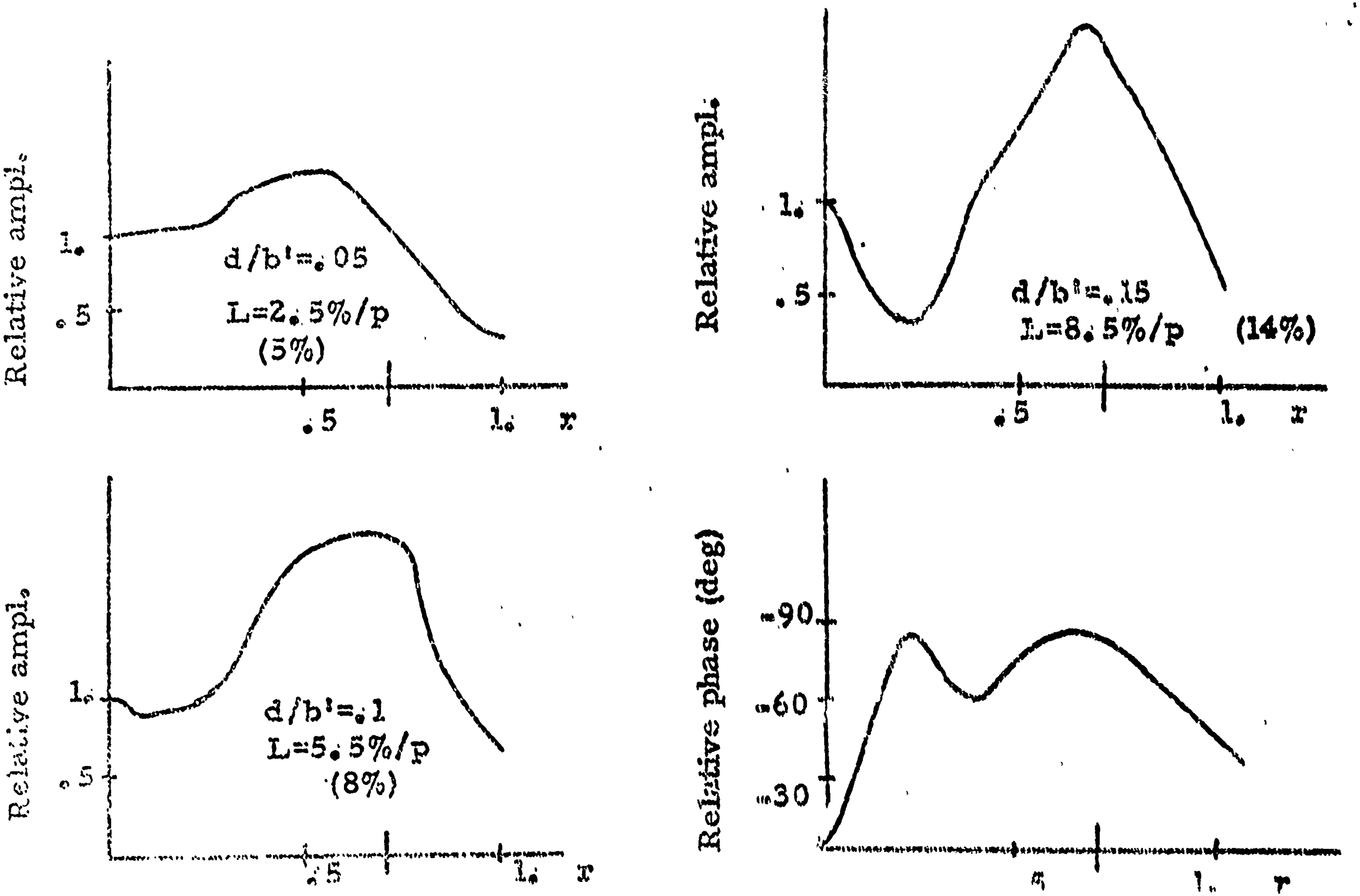

Confocal
d/b=1.
d/b'=-1.
Nl=1.
Relative amplitude
1.
.5
.5
1.
R


NO=.25 N1=2.0 d/b=.2
Rel. ampl.
d/b'=.1
L=.2%/p
.5
1.
r
d/b'=3.
L=7.5%/p
d/b'=1.
L=4%/p
d/b'=4.
L=15%/p
d/b'=2.
L=6%/p
Rel. phase(deg)
20
0
-20
-40

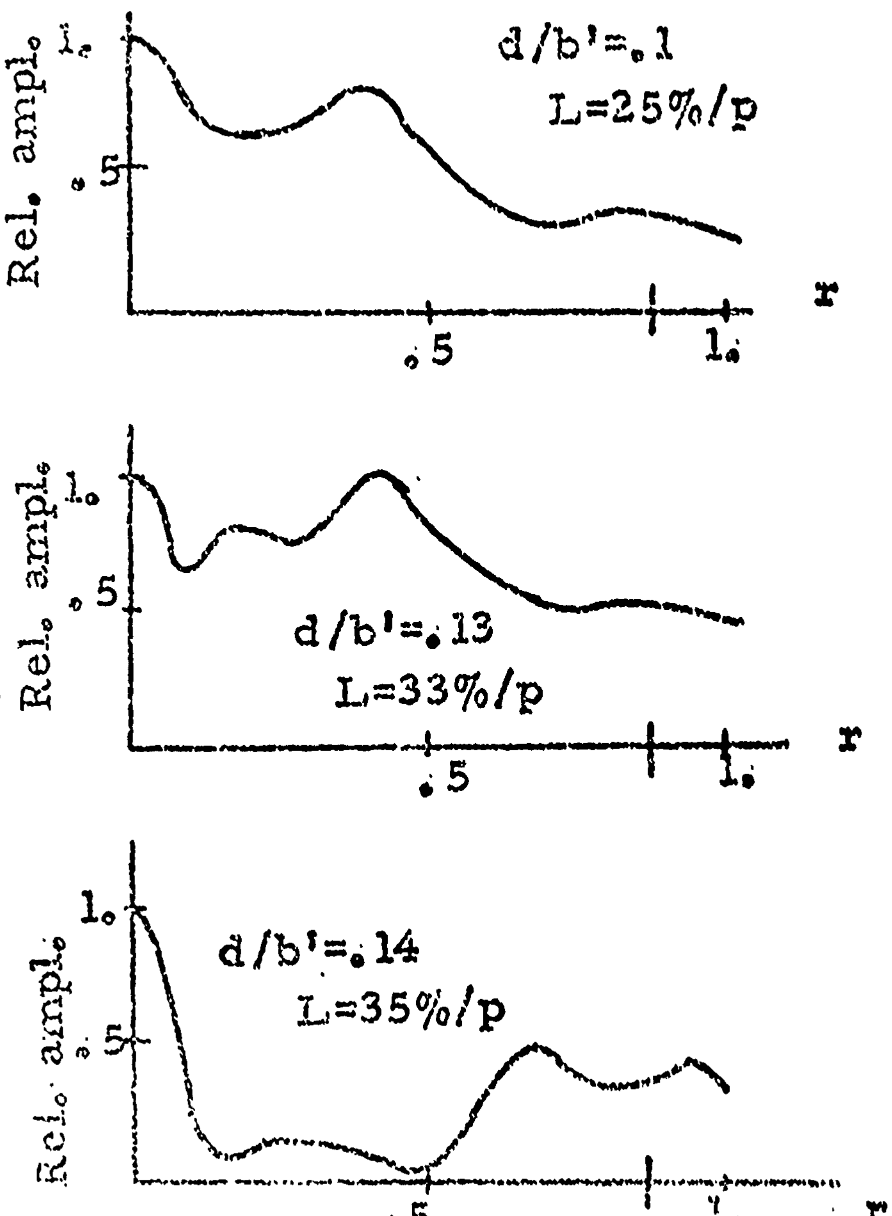
N0=1.5 N1= 2.0 d/b'=.1



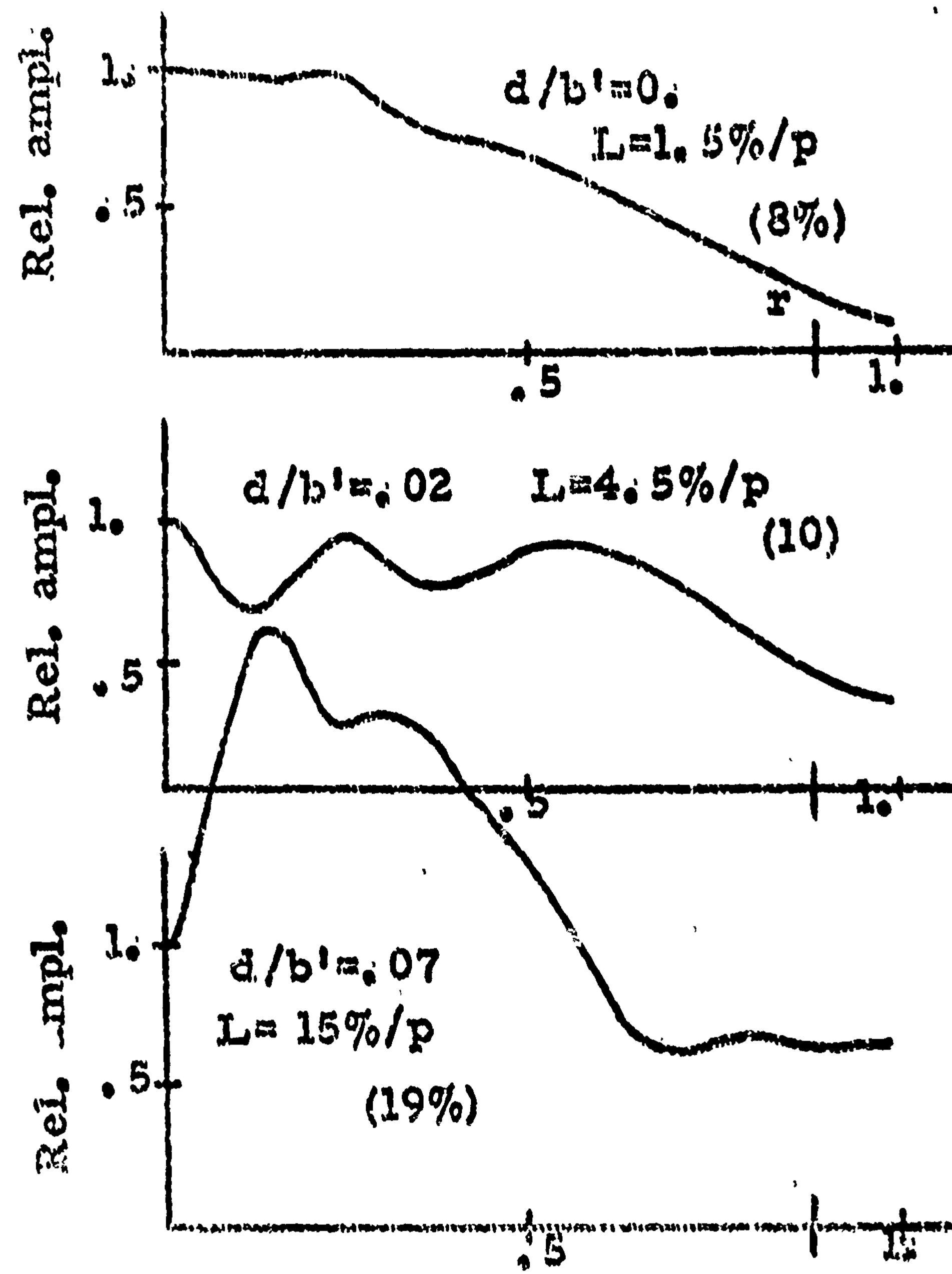
NO = 2.   N1= 4.   d/b=.1


Relative ampl.
1.
.5
d/b'=.05
L=2.5%/p
(5%)
.5
1.
r
Relative ampl.
1.
.5
d/b'=.15
L=8.5%/p
(14%)
.5
1.
r
Relative ampl.
1.
.5
d/b'=.1
L=5.5%/p
(8%)
.5
1.
r
Relative phase (deg)
-90
-60
-30
1.
r

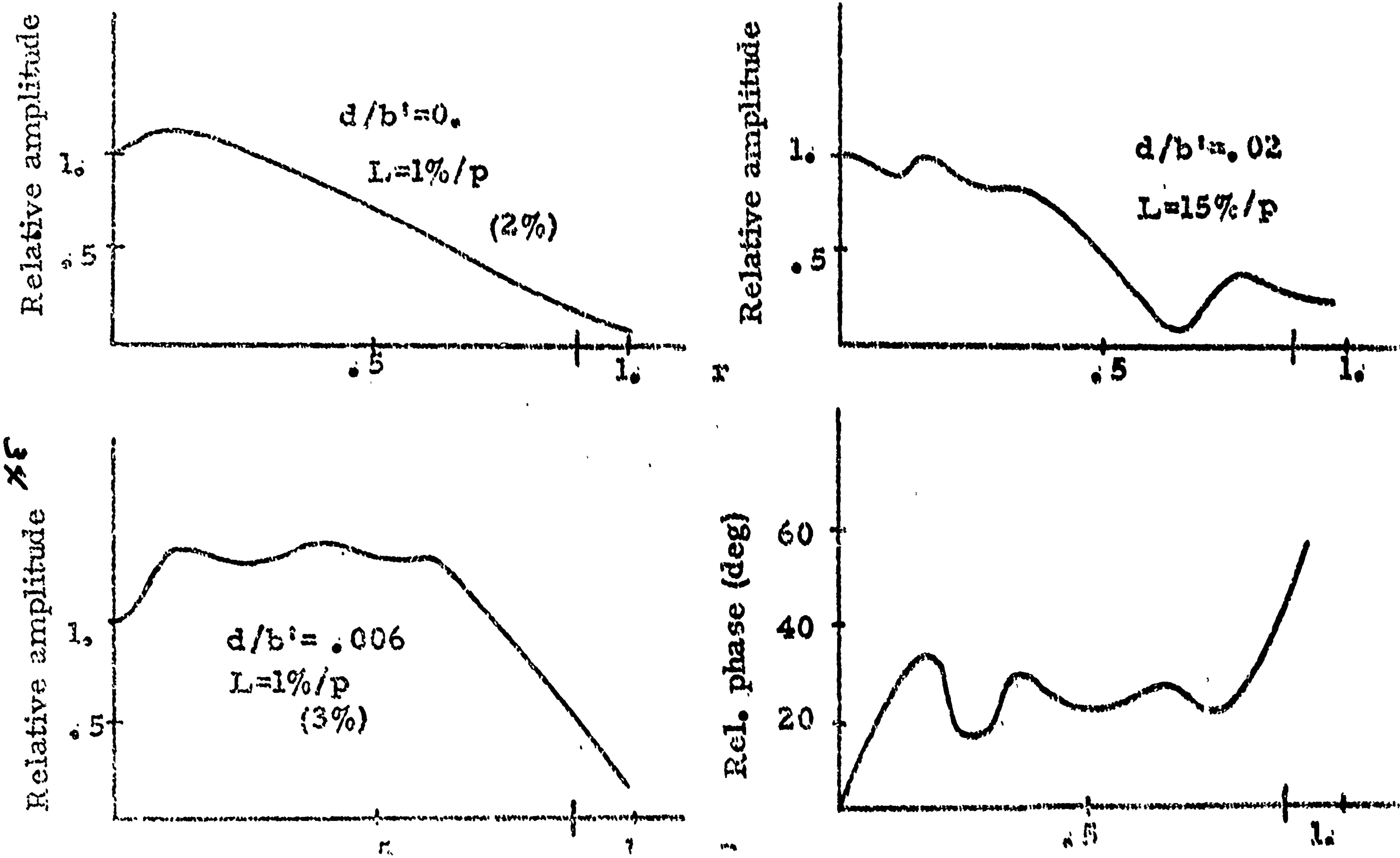
NO= 3. N1= 4. d/b=.1



NO= 3.4 N1= 4.6 d/b=.1



N1= 8.6   d/b=.1



Relative amplitude
1.
.5
d/b'=0.
L=1%/p
(2%)
.5
1.
r
Relative amplitude
1.
.5
d/b'=.02
L=15%/p
.5
1.
Relative amplitude
1.
.5
d/b'=.006
L=1%/p
(3%)
Rel. phase (deg)
60
40
20
.5
1.